Physics (PHZ) 3113
Mathematical Physics

Florida Atlantic University
Fall, 2010

Problem Set I

Due: Tuesday, 31 August 2010

1. (Problems 1.10.5, 9 and 15, p. 22)
   Find the interval of convergence of each of the following power series. Be sure to investigate the endpoints of the interval in each case.

$$S^{(a)} = \sum_{n=1}^{\infty} \frac{x^n}{(n!)^2}, \qquad S^{(b)} = \sum_{n=1}^{\infty} (-1)^n\, n^3\, x^n \qquad \text{and} \qquad S^{(c)} = \sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n}.$$

2. (Problems 1.13.9, 13 and 17, p. 32)
   Expand each of the following functions in a Maclaurin series (a Taylor series about $x = 0$). Write the series as an infinite sum and calculate the first few (non-zero) terms explicitly.

$$f^{(a)}(x) = \frac{1+x}{1-x}, \qquad f^{(b)}(x) = \int_0^x \mathrm{e}^{-t^2}\,\mathrm{d}t \qquad \text{and} \qquad f^{(c)}(x) = \ln\frac{1+x}{1-x}.$$

3. (Problems 1.13.26, 31 and 35, p. 32)
   Find the first few terms of the Maclaurin series for each of the following functions and check your results by computer.

$$f^{(a)}(x) = \frac{1}{\sqrt{\cos x}}, \qquad f^{(b)}(x) = \cos(\mathrm{e}^x - 1) \qquad \text{and} \qquad f^{(c)}(x) = \frac{x}{\sin x}.$$

4. (Problems 1.15.7, 12 and 23c, pp. 40–41)
   Use Maclaurin series to evaluate the following limits.

$$y^{(a)} = \frac{\mathrm{d}^8}{\mathrm{d}x^8}\left(x^6 \tan^2 x\right)\Big|_{x=0}, \qquad y^{(b)} = \lim_{x\to 0} \frac{\tan x - x}{x^3} \qquad \text{and} \qquad y^{(c)} = \lim_{x\to 0}\left(\csc^2 x - \frac{1}{x^2}\right).$$

5. (Problem 1.16.28, p. 42)
   The energy of an electron at speed v in special relativity theory is $E = mc^2(1 - v^2/c^2)^{-1/2}$, where m is the electron mass, and c is the speed of light. The factor mc^2 is called the ***rest mass energy*** (the energy when $v = 0$). Find the first three terms of the series expansion of E in v. What is the second term in the series?

6. (Problem 1.16.33, p. 43)
   If you are at the top of a tower of height h above the surface of the earth, show that the distance you can see along the surface of the earth is approximately $s = \sqrt{2Rh}$, where R is the radius of the earth. (See the figure and the hints in the book.)

7. (Problem 4.12.16, p. 237)
   In kinetic theory, we have to evaluate integrals of the form

$$I(n) = \int_0^\infty t^n \, \mathrm{e}^{-at^2} \, \mathrm{d}t.$$

   Given that $I(0) = \sqrt{\pi/4a}$, evaluate I for all integers n.