Physics (PHZ) 3113 | Florida Atlantic University
Mathematical Physics | Fall, 2010

Problem Set III

Due: Thursday, 16 September 2010

1. (Problems 7.5.2 and 10, pp. 354–355)
   Sketch several periods of the functions whose values on the fundamental interval $-\pi < x < \pi$ are given by the following.

   $$f^{(\mathrm{a})}(x) := \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi/2 \\ 0 & \pi/2 < x < \pi \end{cases} \qquad \text{and} \qquad f^{(\mathrm{b})}(x) := \begin{cases} \pi + x & -\pi < x < 0 \\ \pi - x & 0 < x < \pi. \end{cases}$$

   Expand each in a Fourier sine-cosine series.

2. (Problems 7.7.1 and 7, pp. 360 and 354–355)
   Sketch several periods of the functions whose values on the fundamental interval $-\pi < x < \pi$ are given by the following.

   $$f^{(\mathrm{a})}(x) := \begin{cases} 1 & -\pi < x < 0 \\ 0 & 0 < x < \pi \end{cases} \qquad \text{and} \qquad f^{(\mathrm{b})}(x) := \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi. \end{cases}$$

   Expand each in a Fourier exponential series.

3. (Problems 7.8.12 and 14, p. 363)
   a. Consider the function $f(x) := \mathrm{e}^x$ restricted to the fundamental interval $I := -\pi < x < \pi$, and then extended to the real line again periodically. Sketch several periods of this function and calculate a Foruier series for it.
   b. Repeat the previous part for the interval $I := 0 < x < 2\pi$.
   c. Repeat the previous part for the function $f(x) := \sin \pi x$ on the interval $-1/2 < x < 1/2$.
   d. Repeat the previous part for the interval $0 < x < 1$.

4. (Problems 3.9.23 and 24, p. 371)
   a. If a violin string is plucked, it is possible to find a formula $f(x, t)$ for the displacement at time t of any point x of the vibrating string from its equilibirum position. It turns out that in solving this problem we need to expand the function $f(x, 0)$, whose graph is the initial shape of the string, in a Fourier sine series. Find this series if a string of length l is initially pulled aside a small distance h at its center, as shown in the book.
   b. Now suppose that the string is stopped at the center and only half of it is plucked. Once again, see the figure in the book and calculate the series in this case. Note that $f(x, 0) = 0$ for $l/2 < x < l$.