Physics (PHZ) 3113 Florida Atlantic University
Mathematical Physics Fall, 2010

Problem Set IV

Due: Thursday, 23 September 2010

1. (Problems 7.11.6 and 8, pp. 377–378)
   Use Parseval's theorem and the results of the problems indicated to sum the following series:

   a. $\displaystyle\sum_{n\geq 1} \frac{1}{n^4}$ (see Problem 7.9.9).

   b. $\displaystyle\sum_{\substack{n\geq 1 \\ n \text{ odd}}} \frac{1}{n^4}$ (see Problem 7.9.10).

2. (Problems 7.12.3 and 7, p. 384)
   Find the exponential Fourier transform of

$$f^{(\mathrm{a})}(x) := \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \\ 0 & |x| > \pi \end{cases} \qquad \text{and} \qquad f^{(\mathrm{b})}(x) := \begin{cases} |x| & |x| < 1 \\ 0 & |x| > 1. \end{cases}$$

   Use these results to write each $f(x)$ as a Fourier integral. (That is, write each as a superposition of plane waves in the form of an integral over α.)

3. (Problems 7.12.24 and 33, pp. 385–386)

   a. Find the exponential Fourier transform of $f(x) := \mathrm{e}^{-|x|}$ and write the inverse transform to show that

$$\int_0^\infty \frac{\cos \alpha x}{\alpha^2 + 1}\, \mathrm{d}\alpha = \frac{\pi}{2}\, \mathrm{e}^{-|x|}.$$

   b. Find the Fourier transform of $1/(1 + x^2)$.
   Hint: Interchange x and α in the result of the first part.

   c. Verify Parseval's theorem for the Fourier transform computed in part (a).