Physics (PHZ) 3113 | Florida Atlantic University
Mathematical Physics | Fall, 2010

Problem Set V

Due: Tuesday, 5 October 2010

1. (Problems 2.9.12, 18 and 23, pp. 63–64)
   Express each of the following complex numbers in the Cartesian form $z = x + \mathrm{i}y$.
   $$z^{(\mathrm{a})} := 4\,\mathrm{e}^{-8\pi\mathrm{i}/3}, \qquad z^{(\mathrm{b})} := \left(\frac{1+\mathrm{i}}{1-\mathrm{i}}\right)^4 \qquad \text{and} \qquad z^{(\mathrm{c})} := \frac{(1+\mathrm{i})^{48}}{(\sqrt{3}-\mathrm{i})^{25}}.$$

2. (Problems 2.10.10, 19 and 23, pp. 66–67)
   Find all complex values of each of the following roots.
   $$z^{(\mathrm{a})} := \sqrt[5]{32}, \qquad z^{(\mathrm{b})} := \sqrt[3]{\mathrm{i}} \qquad \text{and} \qquad z^{(\mathrm{c})} := \sqrt[4]{8\mathrm{i}\sqrt{3}-8}.$$

3. (Problems 2.11.4, 7 and 10, p. 69)
   Find all complex values of each of the following functions.
   $$z^{(\mathrm{a})} := \mathrm{e}^{3\ln 2 - \mathrm{i}\pi}, \qquad z^{(\mathrm{b})} := \tan(\mathrm{i}\ln 2) \qquad \text{and} \qquad z^{(\mathrm{c})} := \sin(\mathrm{i}\ln \mathrm{i}).$$

4. (Problems 2.12.2, 3, 8, 12 and 16, p. 71)
   Verify each of the following identities using the definitions of the standard trigonometric and hyperbolic functions of a complex variable $z = x + \mathrm{i}y$.
   a. $\cos z = \cos x \cosh y - \mathrm{i} \sin x \sinh y$
   b. $\sinh z = \sinh x \cos y + \mathrm{i} \cosh x \sin y$
   c. $\cosh 2z = \cosh^2 z + \sinh^2 z$
   d. $\cos^4 z + \sin^4 z = 1 - \frac{1}{2}\sin^2 2z$
   e. $\tan \mathrm{i}z = \mathrm{i} \tanh z$

5. (Problems 2.11.27, 29 and 36, p. 71)
   Evaluate each of the following functions in the Cartesian form $z = x + \mathrm{i}y$
   $$z^{(\mathrm{a})} := \sin(4+3\mathrm{i}), \qquad z^{(\mathrm{b})} := \cosh 2\pi\mathrm{i} \qquad \text{and} \qquad z^{(\mathrm{c})} := \sinh\left(1 + \frac{\mathrm{i}\pi}{2}\right).$$

6. (Problems 2.14.7, 9 and 19, p. 74)
   Find *all* complex values of each of the following functions.
   $$z^{(\mathrm{a})} := \ln \frac{1+\mathrm{i}}{1-\mathrm{i}}, \qquad z^{(\mathrm{b})} := (-1)^{\mathrm{i}} \qquad \text{and} \qquad z^{(\mathrm{c})} := \cos(\pi + \mathrm{i}\ln 2).$$

7. (Problems 2.15.7, 10 and 13, p. 74)
   Find *all* complex values of each of the following inverse functions.

$$z^{(\mathrm{a})} := \tan^{-1}(\mathrm{i}\sqrt{2}), \qquad z^{(\mathrm{b})} := \cos^{-1}\frac{5}{4} \qquad \text{and} \qquad z^{(\mathrm{c})} := \cosh^{-1}(-1).$$

8. (Problems 14.1.8, 12 and 16, p. 667)
   Find the real and imaginary parts $u(x, y)$ and $v(x, y)$, respectively, of the following functions of a complex variable $z = x + \mathrm{i}y$.

$$f^{(\mathrm{a})}(z) := \sin z, \qquad f^{(\mathrm{b})}(z) := \frac{z}{z^2+1} \qquad \text{and} \qquad f^{(\mathrm{c})}(z) := z^2 - \bar{z}^2.$$

9. (Problems 14.2.8, 12 and 16, p. 672)
   Use the Cauchy–Riemann conditions to determine all points z where each of the functions in the previous problem is analytic.

10. (Problems 14.2.36, 40 and 41, p. 667)
    Expand each of the following functions of a complex variable $z = x + \mathrm{i}y$ in a power series about the origin $z = 0$ in the complex plane. Find the disk of convergence for each.

$$f^{(\mathrm{a})}(z) := \sqrt{1+z^2}, \qquad f^{(\mathrm{b})}(z) := \frac{1}{1-z} \qquad \text{and} \qquad f^{(\mathrm{c})}(z) := \mathrm{e}^{\mathrm{i}z}.$$