Physics (PHZ) 3113 Florida Atlantic University
Mathematical Physics Fall, 2010

Problem Set VI

Due: Tuesday, 19 October 2010

1. (Problems 14.3.4 and 12, p. 677)
   Calculate the following line integrals in the complex plane by direct integration, *i.e.*, without using the Cauchy integral theorems.
   $$I^{(\mathrm{a})} := \int_0^{\mathrm{i}\infty} \frac{\mathrm{d}z}{1-z^2} \qquad \text{and} \qquad I^{(\mathrm{b})} := \int_0^{1+2\mathrm{i}} |z|^2 \, \mathrm{d}z.$$
   In the latter case, do the integral twice, once along the straight line joining the two points, and again along the right-angle path $(0,0) \to (0,2) \to (1,2)$. (See the figures in the book.) In the former case, the integral is along the positive imaginary axis.

2. (Problems 14.3.18, 20, and 24, pp. 677–678)
   Use the Cauchy integral formulae to evaluate the contour integrals
   $$J^{(\mathrm{a})} := \oint_{C^{(\mathrm{a})}} \frac{\sin 2z \, \mathrm{d}z}{6z-\pi}, \qquad J^{(\mathrm{b})} := \oint_{C^{(\mathrm{b})}} \frac{\cosh z \, \mathrm{d}z}{2\ln 2 - z} \qquad \text{and} \qquad J^{(\mathrm{c})} := \oint_{C^{(\mathrm{c})}} \frac{\cosh z \, \mathrm{d}z}{(2\ln 2 - z)^5}.$$

3. (Problems 14.4.5 and 7, p. 681)
   Identify all singular points of the functions
   $$f^{(\mathrm{a})}(z) := \frac{z-1}{z^3(z-2)} \qquad \text{and} \qquad f^{(\mathrm{b})}(z) := \frac{2-z}{1-z^2}.$$
   Develop a Laurent series expansion about the origin for each in each annular ring bounded by those singular points. Use the interior expansion to calculate each residue at the origin.

4. (Problem 14.4.10, p. 682)
   For each of the following functions, say whether the indicated point is regular, and essential singularity, or a pole. If it is a pole, say what order it is.
   a. $\dfrac{\mathrm{e}^z - 1}{z^2+4}$ at $z = 2\mathrm{i}$
   b. $\tan^2 z$ at $z = \dfrac{\pi}{2}$
   c. $\dfrac{1-\cos z}{z^4}$ at $z = 0$
   d. $\cos \dfrac{\pi}{z-\pi}$ at $z = \pi$