Physics (PHZ) 3113 | Florida Atlantic University
Mathematical Physics | Fall, 2010

Problem Set VII

Due: Tuesday, 16 November 2010

1. (Problems 14.7.6, 12 and 16, p. 699)
   Evaluate the following definite integrals of real functions by relating them to contour integrals in the complex plane.

$$I^{(a)} := \int_0^{2\pi} \frac{\mathrm{d}\theta}{(2+\cos\theta)^2}, \qquad I^{(b)} := \int_0^{\infty} \frac{x^2\,\mathrm{d}x}{x^4+16} \qquad \text{and} \qquad I^{(c)} := \int_0^{\infty} \frac{x\sin x\,\mathrm{d}x}{9x^2+4}.$$

2. (Problems 14.7.25 and 27, p. 700)
   Consider the definite integrals

$$I^{(a)} := \int_0^{\infty} \frac{x\sin x}{9x^2-\pi^2}\,\mathrm{d}x \qquad \text{and} \qquad I^{(b)} := \int_0^{\infty} \frac{\cos\pi x}{1-4x^2}\,\mathrm{d}x.$$

   Determine whether each integral exists. If it does, evaluate it by relating it to a contour integral. If not, then evaluate its Cauchy principal value by relating that integral to a contour integral.

3. (Problems 14.7.34 and 35, p. 700)
   Evaluate the definite integrals

$$I^{(a)} := \int_0^{\infty} \frac{\sqrt{x}\,\mathrm{d}x}{(1+x)^2} \qquad \text{and} \qquad I^{(b)} := \int_0^{\infty} \frac{x^{1/3}\,\mathrm{d}x}{(1+x)(2+x)}$$

   by relating each to a contour integral around an appropriate "keyhole contour."

4. (Problem 14.7.42, p. 701)
   Let $F(z) := f'(z)/f(z)$, where $f(z)$ is analytic except at isolated points.

   a. Show that the residue of $F(z)$ at an n^{th}-order zero of $f(z)$ is n.
      Hint: If $f(z)$ has a pole of order n at a, then $f(z) = a_n\,(z-a)^n + a_{n+1}\,(z-a)^{n+1} + \cdots$.
   b. Show that the residue of $F(z)$ at a p^{th}-order pole of $f(z)$ is n.
      Hint: See the definition of a pole of order p at the end of Section 14.4.