Physics (PHZ) 3113 — Florida Atlantic University
Mathematical Physics — Fall, 2009

Review Problems I

Recommended Reading: Chow, Appendix 1; Boas, Chapter 1.

1. (Chow A1.7, p. 510) Prove that

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \text{and} \qquad \cos^2 x = \frac{1+\cos 2x}{2},$$

   and that

$$A\cos x + B\sin x = \sqrt{A^2+B^2}\,\sin(x+\delta), \quad \text{where} \quad \tan\delta := \frac{A}{B}$$

   for all real x, A and B.

2. (Chow A1.8, p. 510) Prove that

$$\cosh^2 x - \sinh^2 x = 1 \qquad \text{and} \qquad \operatorname{sech}^2 x + \tanh^2 x = 1$$

   for all real x.

3. (Chow A1.9, p. 511) Calculate the limit of the function $f(x) := x^2$ as $x \to 2$, and show that $f(x)$ is continuous there.

4. (Boas 1.4.6) Evaluate the partial sums

$$S_N := \sum_{n=1}^{N} \frac{1}{n(n+1)},$$

   and show that they converge in the limit $N \to \infty$.
   Hint: Expand the summand using partial fractions.

5. (Boas 1.6.4) Use the comparison test to prove that series

$$S_{(a)} := \sum_{n=1}^{\infty} \frac{1}{2^n+3^n} \qquad \text{and} \qquad S_{(b)} := \sum_{n=1}^{\infty} \frac{1}{n2^n}.$$

   converge.

6. (Boas 1.6.8 and 14) Use the integral test to determine whether the series

$$S_{(a)} := \sum_{n=1}^{\infty} \frac{n}{n^2+4} \qquad \text{and} \qquad S_{(b)} := \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+9}}$$

   converge or diverge.

7. (Boas 1.6.21, 26 and 29) Use the ratio test to determine whether the series

$$S_{(\mathrm{a})} := \sum_{n=0}^{\infty} \frac{5^n (n!)^2}{(2n!)}, \qquad S_{(\mathrm{b})} := \sum_{n=0}^{\infty} \frac{(n!)^3\, \mathrm{e}^{3n}}{(3n)!} \qquad \text{and} \qquad S_{(\mathrm{c})} := \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{(n)!}$$

converge or diverge.

8. (Boas 1.17.1, 4 and 6) Use the alternating series test to determine whether the series

$$S_{(\mathrm{a})} := \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}, \qquad S_{(\mathrm{b})} := \sum_{n=1}^{\infty} \frac{(-3)^n}{n!} \qquad \text{and} \qquad S_{(\mathrm{c})} := \sum_{n=1}^{\infty} \frac{(-1)^n n}{n+5}$$

converge or diverge.

9. Which of the series from the previous problem converge absolutely?

10. (Chow A1.13, p. 520) Use Gauss' test to determine whether the series

$$S := \left(\frac{1}{2}\right)^2 + \left(\frac{1 \times 3}{2 \times 4}\right)^2 + \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^2 + \cdots$$

converges or diverges. Show that neither the ratio test nor Raabe's test would be conclusive for this series.