

Review Problems II

Recommended Reading: Chow, Appendix 2; Boas, Sections 3.1–3.3.

1. (Chow A2.1, p. 540) Solve the system

$$\begin{aligned}2x - y + 2z &= 2 \\ x + 10y - 3z &= 5 \\ -x + y + z &= -3\end{aligned}$$

of linear equations using Cramer's rule.

2. (Chow A2.2, p. 540) Evaluate the determinants

$$D_{(a)} := \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}, \quad D_{(b)} := \begin{vmatrix} 5 & 1 & 8 \\ 15 & 3 & 6 \\ 10 & 4 & 2 \end{vmatrix} \quad \text{and} \quad D_{(c)} := \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix},$$

where in the third case θ is an arbitrary real number.

3. (Chow A2.3, p. 542) Expand the second-order determinant

$$D := \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

using the Laplace development on each of the four rows and columns in the matrix. Show explicitly that one finds the same overall value in each case.

4. (Chow A2.4, p. 542) Evaluate the determinant

$$D := \begin{vmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{vmatrix}$$

using Laplace developments on the first row and on the first column. Show that you get the same result in each case.

5. (Boas 3.3.7) Use the seven rules for determinants enumerated in Chow's book to show that

$$D := \begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (c-a)(b-a)(c-b) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 0 & 1 \end{vmatrix} = (c-a)(b-a)(c-b).$$

Do not simply evaluate the determinants in this problem. The point is to practice using the rules while establishing each equality above.

6. (Boas 3.3.10) A square matrix A is called skew-symmetric if its components obey $A_{ij} = -A_{ji}$. Show that the determinant of a skew-symmetric matrix of odd order must vanish.