Physics (PHZ) 3113 | Florida Atlantic University
Mathematical Physics | Fall, 2009

Review Problems VI

Recommended Reading: Chow, pp. 233–238.

1. (Chow 6.2) Given the complex numbers

$$z_1 := \frac{3+4\mathrm{i}}{3-4\mathrm{i}} \qquad \text{and} \qquad z_2 := \left(\frac{1+2\mathrm{i}}{1-3\mathrm{i}}\right)^2,$$

   find their polar forms, complex conjugates, moduli, product, and quotients.

2. (Chow 6.3) The absolute value or modulus of a complex number $z =: x + \mathrm{i}y$ is defined as

$$|z| = \sqrt{zz^*} = \sqrt{x^2+y^2}.$$

   If z_1 and z_2 are complex numbers, show that:

   a. $|z_1 z_2| = |z_1|\,|z_2|$,
   b. $|z_1/z_2| = |z_1|/|z_2|$ for $z_2 \neq 0$,
   c. $|z_1 + z_2| \leq |z_1| + |z_2|$, and
   d. $|z_1 - z_2| \geq \big||z_1| - |z_2|\big|$.

3. (Chow 6.4) Find all possible complex values of

$$z_{(\mathrm{a})} := \sqrt[5]{-32} \qquad \text{and} \qquad z_{(\mathrm{b})} := \sqrt[3]{1+\mathrm{i}},$$

   and plot them in the complex plane.

4. (Chow 6.5) Use De Moivre's theorem to show that

   a. $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$, and
   b. $\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$.