Course Outline

Date	Lecture Topic	Reading
	• **Graduate Mathematical Physics**	
Tue 24 Aug	• **Linear Algebra: Theory** • Vectors, bases and components • Linear maps and dual vectors • Inner products and adjoint operators • Direct sums and quotients	**744 – 756**
Thu 26 Aug	• **Linear Algebra: Applications** • *Problem set I available* • Ex. A.2 (747), A.3–5 (753), A.8 (756), A.9 (762), A.15 (770) • Linear systems of equations • Matrices and determinants • Eigenvalues and diagonalization • Jordan normal form	**757 – 772**
Tue 31 Aug	• **The Calculus of Variations** • Functionals and their variations • The Euler–Lagrange equations • Lagrangian mechanics • Noether's theorem and gauge theory	**1 – 17** ©
Thu 02 Sep	• **Fields and Continuum Mechanics** • Many degrees of freedom • Continuum limit and mechanics of media • Maxwell theory and gauge fields • Fluid mechanics	**17 – 26** ©
Tue 07 Sep	• **Advanced Topics in Variational Calculus** • *Problem set I due, problem set II available* • Ex. 1.2 (38), 1.8 (43), 1.13 (46) • Pr. 1.6 (41), 1.12 (45) • Problems with variable endpoints • Constraints and Lagrange multipliers • The second variation • Rayleigh–Ritz problems	**27 – 38** ©
Thu 09 Sep	• **Function Spaces** • Functions as vectors • Convergence and Hilbert space • Completeness and Hilbert bases	**50 – 62**

Date	Lecture Topic	Reading
	• Best approximation and Parseval's theorem	
Tue 14 Sep	• **Fourier Series and Transforms** • Fourier series and their limits • Fourier transforms • Gibbs' phenomenon • The Poisson summation formula	**779 – 795**
Thu 16 Sep	• **Linear Operators and Distributions** • *Problem set II due, problem set III available* • Ex. B.1 (790), B.3 (790), B.6 (792), 2.3 (64), 2.5 (65), 2.13 (78), 2.20–22 (84) • Orthogonal polynomials • Linear operators • Test functions and distributions • Calculus with distributions	**62 – 75**
Tue 21 Sep	• **Linear Ordinary Differential Equations** • Existence and uniqueness of solutions • Linear independence and the Wronskian • Normal form and singular points • Solution of inhomogeneous equations	**86 – 98**
Thu 23 Sep	• **Linear Ordinary Differential Operators** • Operators, domains and boundary conditions • Adjoint operators and boundary conditions • Self-adjoint problems and extensions • Introduction to the eigenvalue problem	**101 – 116**
Tue 28 Sep	• **Completeness of Eigenfunctions** • *Problem set III due, problem set IV available* • Ex. 3.3 (99), 4.2 (108), 4.4 (111) • Pr. 3.4 (99), 4.13 (136) • Operators with discrete spectrum • Rayleigh–Ritz and other methods • Operators with continuous spectrum • Generalized eigenfunctions	**117 – 131**
Thu 30 Sep	• **Introduction to Green Functions** • The Fredholm alternative • Theory and methods of Green functions • Two-point and initial-value problems • The modified Green function	**140 – 150**
Tue 05 Oct	• **Applications of Green Functions** • Hermiticity and Lagrange's identity	**150 – 159**

Date	Lecture Topic	Reading
	• Eigenfunction expansions • Inhomogeneous boundary conditions • Causality and analyticity	

Thu 07 Oct	• **Analytic Properties of Green Functions**
	• *Problem set IV due, problem set V available*
	  • Ex. 5.1 (167), 5.2 (168), 5.5 (169), 5.7 (171)
	  • Pr. 5.9 (172)
	• Causality and analyticity revisited
	• Plemelj formulae and principal values
	• Resolvent operators and Green functions
	• Locality and Green functions
		155 – 167

Tue 12 Oct	• **Introduction to Partial Differential Equations** • Classification of partial differential equations • Characteristics and Cauchy data • First-order equations • The wave equation in two dimensions	**174 – 185**
Thu 14 Oct	• **The Wave Equation** • The d'Alambert and Fourier solutions • The retarded Green function • Waves in odd vs. even dimensions • Huygens' principle	**181 – 195**

Tue 19 Oct	• **The Heat Equation**
	• *Problem set V due, problem set VI available*
	  • Ex. 6.2 (184), 6.3 (185), 6.15 (225)
	  • Pr. 6.13 (224), 6.14 (224)
	• The heat kernel
	• The causal green function
	• Duhamel's principle
	• The Schrödinger equation
		196 – 201
		©

Thu 21 Oct	• **The Laplace Equation** • The Poisson and Laplace equations • Dirichlet and Neumann problems • Existence and uniqueness of solutions • Separation of variables	**201 – 213**
Tue 26 Oct	• **The Poisson and Helmholtz Equations** • Eigenfunction expansions and Green functions • Boundary value problems • Method of images	**213 – 223**

Date	Lecture Topic	Reading
	• Monochromatic waves	
Thu 28 Oct	• **Dispersion and Resonance** • *Problem set VI due, problem set VII available* • Ex. 6.9 (218), 6.10 (219), 6.16 (225), 6.17 (226), 7.3 (260) • Pr. 6.12 (223), 7.3 (260) • Dispersive waves • Phase vs. group velocity • Wakes and rays • Rayleigh's equation	**231 – 246**
Tue 02 Nov	• **Spherical Harmonics** • Calculus in curvilinear coordinates • Separation of variables in spherical coordinates • Legendre polynomials • General spherical harmonics	**264 – 278**
Thu 04 Nov	• **Cylindrical Bessel Functions** • Bessel's equation and its solutions • Recursion relations and other identities • Orthogonality and Hankel transforms • Modified Bessel functions	**278 – 293**
Tue 09 Nov	• **Spherical Bessel Functions** • *Problem set VII due, problem set VIII available* • Ex. 8.1 (274), 8.3 (287), 8.5 (303), 8.6 (303), 8.11 (307), 8.14 (310) • The spherical Bessel equation • Recursion relations and other identities • Singular endpoints and regularity conditions • Weyl's theorem	**294 – 305**
Thu 11 Nov	• **(No Class due to Veterans' Day)**	
Tue 16 Nov	• **Integral Transforms** • Introduction to integral equations • Fourier transforms • Laplace transforms • Radon transforms	**311 – 321**
Thu 18 Nov	• **Exact Solution of Integral Equations** • Separable kernels and the eigenvalue problem • Inhomogeneous problems • Singular integral equations and principal parts • Wiener–Hopf equations	**321 – 332**

Date	Lecture Topic	Reading
Tue 23 Nov	• **Approximate Methods for Integral Equations** • *Problem set VIII due, problem set IX available* • Ex. 9.2 (343), 9.3 (343), 9.5 (344), 9.7 (345), 9.9 (346) • Integral equations and functional analysis • Geometry of operators in Hilbert space • The Born approximation • The Fredholm series	**332 – 342**
Thu 25 Nov	• **(No Class due to Thanksgiving Recess)**	
Tue 30 Nov	• **Non-Linear Waves and Solitons** • Non-linear wave phenomena • Shocks • Weak solutions • Solitons	**246 – 259**
Thu 02 Dec	• **Final Exam** • *Problem set IX due, final exam available*	
Tue 07 Dec	• **Final Exam** • *Final exam due*	