Fall 2010

Mathematical Physics

Physics (PHZ) 5115
3:30 – 4:50 TΘ
Forkas Alumni Center 104

Course Web Site http://physics.fau.edu/~cbeetle/Fall10/PHZ5115/

Instructor

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Office hours: 11:00 – 12:00 TΘ
(or by appointment).

Required Text

- M. Stone and P. Goldbart. *Mathematics for Physics: A Guided Tour for Graduate Students*. (Cambridge, 2009.) ISBN 0-521-85403-0

The theory of functional analysis has become increasingly important to theoretical physics over the past century. Unfortunately, many contemporary mathematical treatments of this subject obscure its conceptual foundations and connection to practical problems of the sort that physicists routinely encounter. This is a pity both because the mathematics offers considerable insight into the structure of physical problems and because physical intuition can shed considerable light on why abstract mathematical ideas are defined as they are and how they relate to one another.

This course aims to bridge the gap between the mathematical theory and its physical applications. We will study functional analysis as a means of gaining insight into practical problems in theoretical physics. Much of the course will center on the analysis of linear partial differential equations. Many students in the course will likely have had considerable experience solving such equations as undergraduates. However, this course is not concerned solely with solving equations, but also with techniques that allow one to discern whether solutions exist, and how many, *before* actually attempting to find one. These techniques help considerably to guide one's thinking when solving practical problems either analytically or numerically. They are also interesting in that a deeper understanding of the mathematics sheds considerable light on the essential conceptual content of associated physical models. Finally, a sound understanding of this material will benefit students as linear differential equations arise repeatedly in almost every course in the graduate physics curriculum.

Mathematical Background and Content

There are several mathematical methods that students in this course ought to have mastered already. These include:

- linear algebra and eigenvalue problems in Hilbert space,
- solution of ordinary differential equations,
- Fourier transforms and the Dirac delta function, and
- separation of variables for the Laplacian in 2 and 3 dimensions.

We will review some of this material at the beginning of the course. However, without prior background in these subjects, students may struggle in the later parts of the course. Any student who is concerned about his or her mathematical preparation should consult with the instructor at his or her earliest opportunity.

The course will cover several mathematical topics that will be useful in other graduate physics courses. We focus on those that are useful in multiple other courses, and in research. These include

- the calculus of variations with applications,
- integral transforms and generalized functions,
- Sturm–Liouville theory and special functions,
- boundary-value problems and Green functions,
- the partial differential equations of mathematical physics, and
- integral equations and their applications.

The course will follow loosely a traditional lecture format, with an emphasis on solving specific problems. Computer demonstrations will be used where possible.

Course Schedule

The following is a rough schedule of the major topics to be covered, and the rough amount of time to be devoted to each:

- Review of Linear Algebra (1 week)
- Calculus of Variations (1½ weeks)
- Hilbert Space and Distribution Theory (1½ weeks)
- Ordinary Differential Equations (1½ weeks)
- The Green Function Method (1½ weeks)
- Partial Differential Equations I (1½ weeks)
- Partial Differential Equations II (1½ weeks)
- Legendre and Bessel Functions (1½ weeks)
- Integral Transforms and Equations (2 weeks)

Each of these will be the subject of one set of homework problems, which will be due roughly every ten days. In addition, there will be one mid-term exam covering the first five topics, each of which may be a review to some extent for some students. The final exam will cover the last four topics.

A note on complex variables: Regrettably, it is not possible to cover all of the material that really ought to be covered in this course in the time allowed. Based primarily on the quality of the discussion in the course text, the methods of complex variables are not part of the syllabus this year. These methods are very important, of course, but so are the topics that are planed for the course. Students interested in a brief introduction to complex variables may like to attend the undergraduate mathematical physics course during the second half of

September this semester. That course will cover complex variables to some extent, albeit with somewhat less rigor than a proper, graduate-level discussion would have.

Grading and Related Policies

Grades in this course will be calculated, via a curve, based on students' performance on

- 45% — nine weekly problem sets. Each problem will be graded pass/fail based on a quick assessment of whether the student has made a serious attempt to solve it.
- 10% — class and/or office-hour participation. The best way to learn the material covered in this course is to devote sufficient time to the homework problems. These problems will not be easy, and students actively engaged in the course will almost certainly have questions. This small portion of the overall grade is intended to reward those students who, in the instructor's opinion, are most aggressively trying to internalize the material.
- 25% — one mid-term exam. This will be an oral exam conducted at the instructor's whiteboard. The exam will consist of a discussion, led by the student, of one or more problems of the instructor's choice from the first five homework problem sets. The schedule of exams will be announced on Tuesday, October 19, with the exams themselves to take place throughout the following week.
- 20% — one final exam. This will be an oral exam conducted at the instructor's whiteboard. The exam will consist of a discussion, led by the student, of one or more problems of the instructor's choice from the last four homework problem sets. The schedule of exams will be announced on Tuesday, November 30, with the exams themselves to take place throughout the following week.

All coursework must be submitted at the beginning of class on the date it is due.

Late Homework and Extra Credit Policies

All assigned coursework must be submitted at the beginning of the lecture on the date it is due. However, if a student has a family emergency or illness, he or she may request an individual extension *by email at least twenty-four hours in advance* of the the lecture during which the assignment is due. The instructor may approve or deny such requests at his discretion.

No extra credit will be offered.

Collaboration on Assigned Coursework

Students are encouraged to collaborate while solving practice problem sets and homework problems. However, once these problems have been discussed and their solutions outlined, each student should prepare his or her own solution. Copying is not allowed.

Important Dates

- Tuesday, October 19 — Mid-Term Exam Schedule Announced
- Thursday, October 21 through Tuesday, October 26 — Mid-Term Exams
- Thursday, November 11 — No Class due to Veterans' Day Holiday
- Thursday, November 26 — No Class due to Thanksgiving Recess
- Tuesday, November 30 — Final Exam Schedule Announced
- Thursday, December 2 through Tuesday, December 7† — Final Exams

Students with Disabilities

In compliance with the Americans with Disabilities Act (ADA), students who require special accommodations due to a disability to properly execute coursework must register with the Office for Students with Disabilities (OSD) located in

Boca Raton — SU 133 (561.297.3880), in
Davie — MOD I (954.236.1222), in
Jupiter — SR 117 (561.799.8585), or at the
Treasure Coast — CO 128 (772-873-3305),

and follow all OSD procedures.

Honor Code

Students at Florida Atlantic University are expected to maintain the highest ethical standards. Academic dishonesty, including cheating and plagiarism, is considered a serious breach of these ethical standards, because it interferes with the University mission to provide a high quality education in which no student enjoys an unfair advantage over any other. Academic dishonesty is also destructive of the University community, which is grounded in a system of mutual trust and places high value on personal integrity and individual responsibility. Harsh penalties are associated with academic dishonesty. For more information, see:

http://www.fau.edu/regulations/chapter4/4.001_Honor_Code.pdf.

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† A date which will live in infamy!