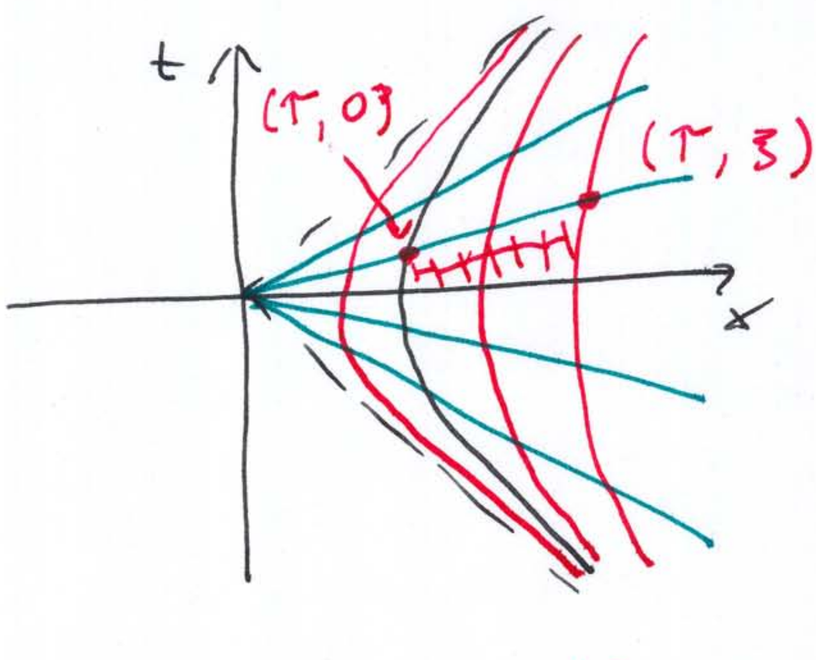
The metric in accelerating radio coordinates:

$$t = \frac{c}{g} e^{g\xi/c^2} \sinh\left(\frac{g\tau}{c}\right)$$

$$x = \frac{c^2}{g} e^{g\xi/c^2} \cosh\left(\frac{g\tau}{c}\right)$$

$$ds^2 = -c^2 dt^2 + dx^2 = e^{2g\xi/c^2}\left(-d\tau^2 + d\xi^2\right)$$

↑ exercise!



$$x(\xi) = \int_0^\xi ds = \int_0^\xi e^{g\xi/c^2}\, d\xi$$

$$dx = e^{g\xi/c^2}\, d\xi \qquad = \frac{c^2}{g}\left(e^{g\xi/c^2} - 1\right)$$

$$\xi(x) = \frac{c^2}{g} \ln\left(1 + \frac{g}{c^2} x\right)$$

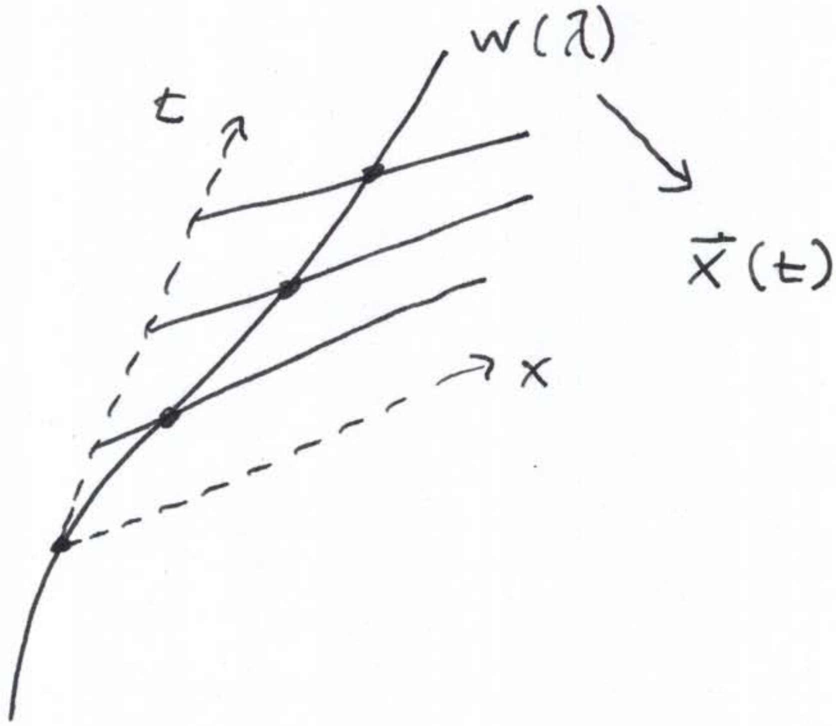
$$d\xi = e^{-g\xi/c^2}\, dx$$

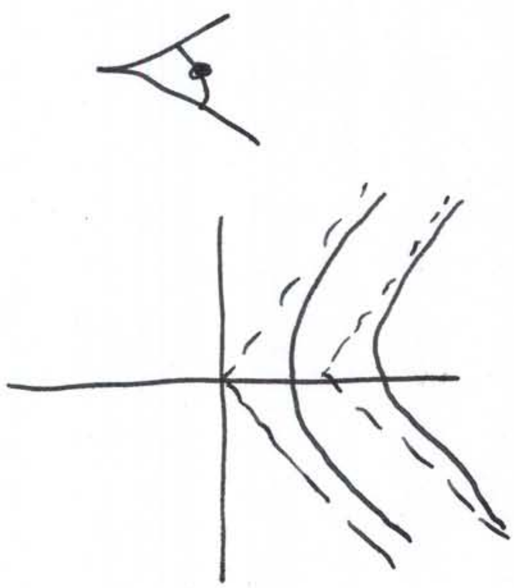
then

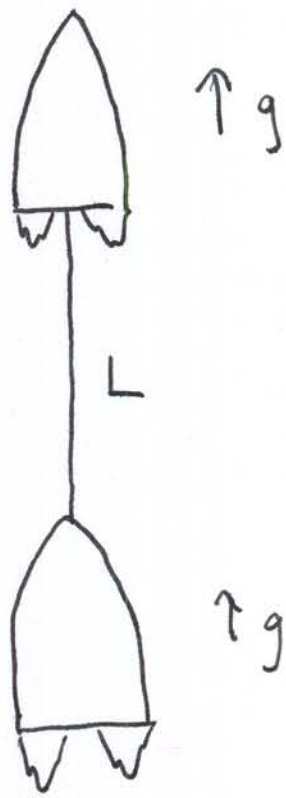
$$ds^2 = -e^{2g\xi/c^2} c^2 d\tau^2 + dx^2$$

$$\boxed{ds^2 = -c^2(1 + gx/c^2)\, d\tau^2 + dx^2}$$

gx is the Newtonian gravitational potential in a uniform field.







$v(t=0) = 0$